Multi-Dimensional Cauchy Method and Adaptive Sampling for an Accurate Microwave Circuit Modelling

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Abstract

The paper presents an effective generic approach for CAD design of microwave circuits. We extend the one-dimensional Cauchy method for frequency response interpolation to a multi-dimensional Cauchy interpolation with respect to both frequency and physical dimensions. The paper also demonstrates the feasibility of applying adaptive sampling to the multi-dimensional rational function expansion. Two examples are given to verify the validity of the proposed approach.

1 Introduction

A number of software packages are now commercially available for Electro-Magnetic (EM) simulation of microwave circuits. These packages, however, are typically very computation intensive and exceed the capabilities of today's computer workstations. Over the past years, there has been a strong interest to circumvent this problem using neural networks [1], space mapping [2] and parameter extraction [3]. The use of Cauchy method has been also proposed in [4][5]. The Cauchy method yields a surprisingly accurate match of the computed points between (interpolated) and even exterior (extrapolated) to the sampled points with the exact solution. The method also allows an easy application of adaptive sampling [8].

Most of the papers published, however, on application of Cauchy method deal with one-dimensional interpolation, namely frequency response interpolation. In this paper, we extend the one-dimensional interpolation for frequency response interpolation to multi-dimensional Cauchy interpolation with respect to both frequency and geometrical dimensions. Two different approaches are suggested to achieve a multi-dimensional approach: A recursive one-dimensional application of the standard Cauchy method and multi-dimensional rational function expansion.

Adaptive sampling can also be applied to the multi-dimensional Cauchy method. Using the recursive method, the samples must lie on a –not necessary uniform– grid and adaptive sampling can only be used in one dimension without constraints.

2 One-Dimensional Cauchy Method

A closer look on system transfer functions, e.g. return loss, insertion loss, tells us that most of these functions can be represented by a rational polynomial in the form

$$S(f) = \frac{a_0 + a_1 f + a_2 f^2 + \cdots}{1 + b_1 f + b_2 f^2 + \cdots} = \frac{a_0 + \sum_{j=1}^{N} a_j f^j}{1 + \sum_{j=1}^{D} b_j f^j}. \quad (1)$$

This scheme is called rational function interpolation or Cauchy method. Consequently, using rational functions as interpolation functions, yields a much closer representation of the systems response than other schemes, e.g. splines.

The efficiency of the Cauchy method is shown in comparison with linear and spline interpolation in Figure 1. The response of a four-pole filter has been sampled at 20 frequency points. Clearly, the Cauchy method is the only interpolation scheme that interpolates the unknown frequency points correctly.

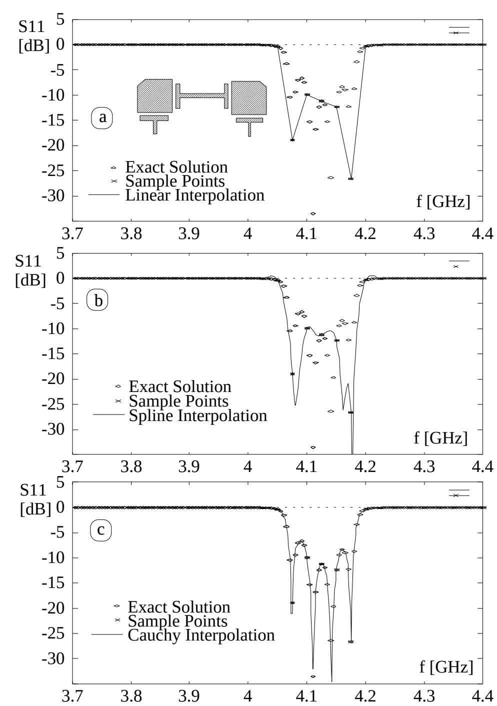

Figure 1: Interpolation of filter response, a) linear, b) cubic spline, c) Cauchy

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DHAENE et al. [8] showed that an adaptive sampling scheme can be applied in the Cauchy Method in order to reduce the number of sampling points to the minimum.

3 Multi-Dimensional Cauchy Method

The rational function interpolation can be extended to the interpolation of multi-dimensional functions. Two new approaches are shown here: A multi-dimensional recursive Cauchy method and a multi-dimensional rational function expansion.

3.1 Recursive Cauchy Method

The recursive method solves the multi-dimensional interpolation using a recursive algorithm. The algorithm itself performs a one-dimensional Cauchy interpolation as described in Section 2. From a given set $\mathcal{H}$ of γ sample points and an arbitrary point p^* the algorithm $\mathbf{C}$ calculates the interpolated function value $S^*(p^*)$.

The set $\mathcal{H}$ is put together by the pairs of sampling points p' to p^γ and their function values S' to S^γ. Thus $\mathcal{H}$ can be written as

$$\mathcal{H} = \{(p', S'), (p'', S''), (p''', S'''), \ldots, (p^\gamma, S^\gamma)\} \qquad (2)$$

The algorithm C can be defined as a function $\mathbf{C}(p^*, \mathcal{H})$ which yields the interpolated response S^* for p^* using the samples $\mathcal{H}$.

$$\mathbf{C}(p^*, \mathcal{H}) \doteq \left[\left(p^*, \{(p', S'), (p'', S''), \ldots (p^n, S^n)\}\right) \to S^*\right] \qquad (3)$$

Using these definitions the algorithm can now be extended to multi-dimensional interpolation. For this purpose the set of sample points must be extended from the one-dimensional sample point set $\mathcal{H}$ to a multi-dimensional sample point array.

3.1.1 Choice of Sample Points in Parameter Space

The sample points in an n-dimensional parameter space are now represented by vectors $\vec{p} = (p_1, p_2, \ldots, p_n)$. For the recursive algorithm the set of sample points $(\vec{p}, S)$ must fall on a complete filled grid of points. Exemplary sample locations in the parameter space are shown in Figure 2 for two and three parameters.

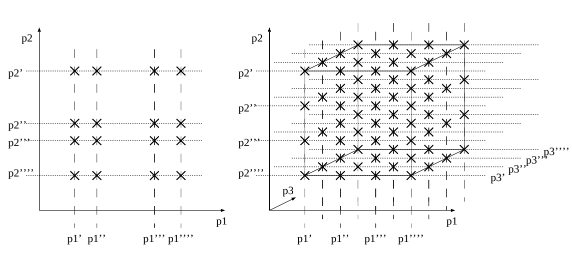


Figure 2: Sample locations for two and three dimensional parameter space

3.1.2 Algorithm Implementation

The goal is to interpolate the function value S^* of an arbitrary located point $\vec{p}^* = p_1^*, p_2^*, \ldots, p_n^*$. The algorithm can be divided into three steps:

Step 1: The root process starts with interpolating the point p^* with constant $p_2 = p_2^*, p_3 = p_3^*$ etc. parallel to the p_1-axes, shown as a dashed line in Figure 3(a). This is a one-dimensional interpolation, so the algorithm $\mathbf{C}$ defined in eqn.(3) can be used. This step yields the desired interpolated point

$$S^* = \mathbf{C}\left([p_1^*, p_2^*, \ldots], \mathcal{A}\right) \qquad (4)$$

where $\mathcal{A}$ is the set of sampling points for p_1' to p_1'''' and $p_2 = p_3, \ldots$=const. These are the points marked with ◍ in Figure 3. They may not fall on the grid of known sample points. If that is the case, the algorithm proceeds to step 2 in order to determine the points ◍. Otherwise (the points are known) the algorithm proceeds with Step 3.

Step 2: The algorithm calls itself for each of the unknown points ◍. In the example the algorithm starts four new child processes

$$\begin{aligned} &\mathbf{C}((p_1', p_2^*), \mathcal{B}_1)\ , \\ &\mathbf{C}((p_1'', p_2^*), \mathcal{B}_2)\ , \\ &\mathbf{C}((p_1''', p_2^*), \mathcal{B}_3)\ , \\ &\mathbf{C}((p_1'''', p_2^*), \mathcal{B}_4) \end{aligned} \qquad (5)$$

as shown in Figure 3(b). The $\mathcal{B}$'s are sets of sample points with a fixed value for p_1 as seen from Figure 3. The interpolation is now performed along the p_2-axis. The routine called is exactly the routine already used in Step 1. The algorithm is thus *recursive*. Again, each subprocess checks if the set $\mathcal{B}$ is from known samples. If not, the algorithm starts another instance of subprocesses in order to interpolate the points included in $\mathcal{B}$ using the next higher dimension. In the example this would be p_3.

Step 3: In case the subprocess determines that all sample points are known, it calculates the interpolated point and hands it back to the parent process which requested that point.

Finally, the answer for the root process eqn.(4) will be found.

Adaptive sampling can only be applied in the last dimension, as all other samples must fall on the grid. This is not a major limitation, because most parameters show a slow variation and the most non-linear parameter can be chosen last.

3.2 Multi-Dimensional Rational Function Expansion

The multi-dimensional method can also be implemented by a single multi-dimensional rational polynomial. in the form

$$S(p_1, p_2, p_3, \ldots) = \frac{P_{num}(p_1, p_2, p_3, \ldots)}{P_{den}(p_1, p_2, p_3, \ldots)} \qquad (6)$$

where $P_{num}(p_1, p_2, p_3, \ldots)$ and $P_{den}(p_1, p_2, p_3, \ldots)$ are arbitrary polynomials of the parameters in the numerator and denominator p_1 to p_n, respectively. Using this approach the coefficients of eqn.6 are determined directly. SAKATA [9] showed the extension into two dimension and the general scheme is discussed shortly here.

One possible expansion of the polynomial for two dimensions could be

$$P(p_1, p_2) = a_0 + a_1 p_1 + a_2 p_2 + a_3 p_1 p_2 + a_4 p_1^2 + a_5 p_2^2 + a_6 p_1 p_2 + \ldots \qquad (7)$$

This approach yields a linear equation system. Solving this system determines directly the coefficients of eqn.7 and, hence, a closed-form and differentiable equation of the system's response $S(p_1, p_2, p_3, \ldots)$.

As there are no restrictions whatsoever, a multi-dimensional adaptive sampling of the parameter space can be applied in analogy to the one-dimensional case.

It should be, however, mentioned, that for large dimensional problems it would be more efficient to split the problem into several problems of lower order, which are then solved recursively as shown in Section 3.1.

4 Examples

Two examples are shown here to demonstrate the interpolation using multi-dimensional Cauchy method.

4.1 Microstrip Line Impedance

The multi-dimensional rational function expansion is demonstrated by modelling the line impedance of a microstrip line respect to the line's width-to-height-ratio $\frac{w}{h}$ and the relative dielectric constant ϵ_r of the substrate.

The modelling algorithm described in Section 3.2 returns a closed-form, differentiable rational function:

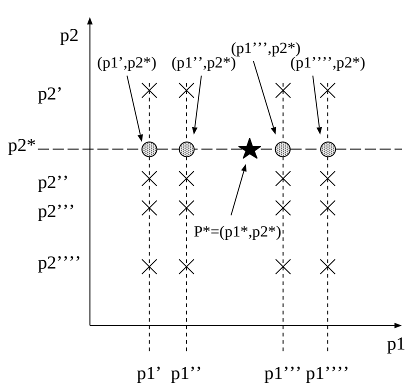
$$Z(p_1 = \epsilon_r, p_2 = \frac{w}{h}) = \qquad (8)$$

$$\frac{549.6232 + 117.0078 p_1 + 9799.3839 p_2 - 7.4367 p_1^2 + \ldots}{1.0 + 0.45021 p_1 + 36.5709 p_2 - 0.01946 p_1^2 + 12.5519 p_2^2 + \ldots}$$

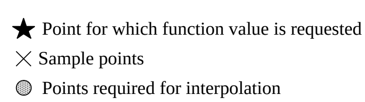
The samples are determined by an adaptive sampling technique similar to the method used in [8]. taking 19 samples. The model provides an accuracy within 0.1% error.

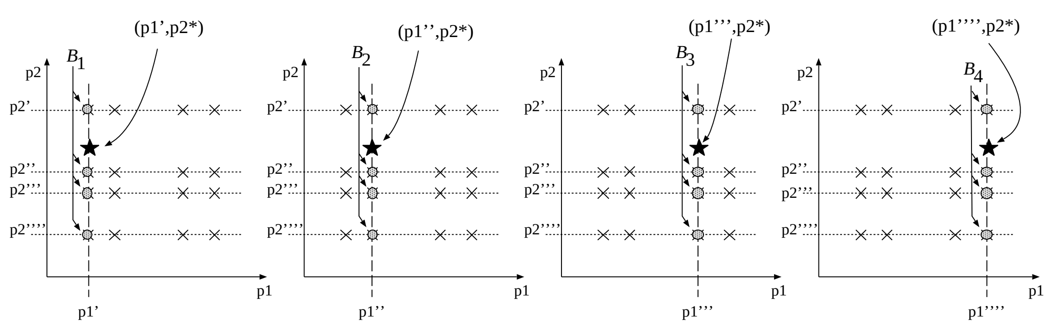
4.2 Narrow-Band 3-Pole Filter

In this example the S-parameters of the response of a planar superconductive microstrip filter as shown in Figure 4 are interpolated. The five parameters are the four geometrical parameters, namely the gap length and resonator lengths, and the frequency. A sample grid with 5 sample points per direction for the geometrical parameter is used.



(a) Step 1





(b) Step 2

Figure 3: Steps in recursive Cauchy method

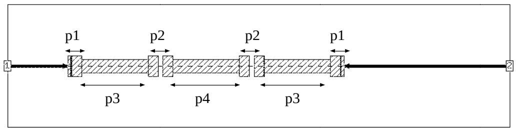


Figure 4: Layout of 3-pole filter withparameters p1 to p4

Due to the underlying EM-simulation software the parameter values are forced to lie on a 1.75 mil grid. The frequency dependency is determined by adaptive sampling, as the last stage of the recursive algorithm.

Shown in Figure 5 is one interpolated response for parameter values not falling on the 1.75 mil grid. For reference, the exact solution, obtained by a finer meshing and finer frequency stepping, is shown, too. One notice the very good agreement from the interpolated response with the exact solution.

In Figure 6 the response is shown, when the multi-dimensional Cauchy method is not applied. Due to the restrictions, that all geometrical values have to fall on the 1.75 mil grid the parameter values must be snapped to the next grid-point above or below.

In addition, the frequency resolution is lost, as only the sampled frequencies can be shown. As a result the filter response degrades to a meaningless shape.

5 Conclusion

In the past many publications have shown the remarkable reduction of computational cost when Cauchy method and adaptive sampling is applied to the frequency response interpolation.

This paper has shown that the method can be extended to the application on multi-dimensional problems. This can either be done by recursive application of the Cauchy method or by an all-in-one multi-dimensional rational polynomial approach. In doing so similar savings of computational expenses for multi-dimensional problems can be achieved as for the one-dimensional case.

Two examples have been given. The first demonstrates that a complete and accurate numerical model of a three-pole filter with 4 geometrical parameter plus frequency dependency can be obtained. The frequency variable determined in the last recursive level was sampled adaptively. The second example illustrates the concept of the multi-dimensional rational function expansion approach. Both approaches can be combined to tackle large-dimensional problems using adaptive sampling for several parameters at the same time.

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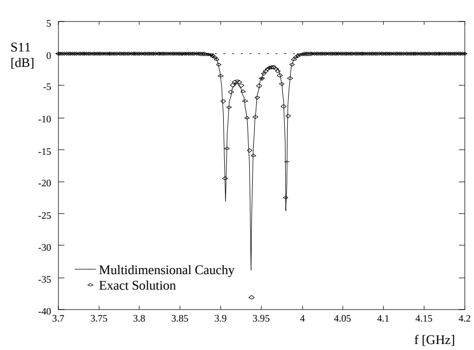

Figure 5: Interpolated S_{11}-response of 3-pole filter

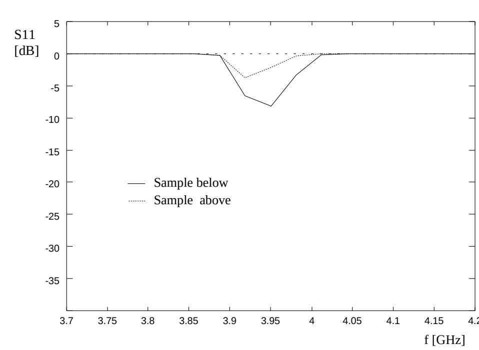

Figure 6: Response using sampled points above and below